Unit 5 Summary

| Prior Learning	Grade 8, Unit 5	Later in Grade 8	High School
Grade 7 ● Volume of prisms ● Area of circles Grade 7, Unit 3 ● Linear relationships	● Define functions. ● Represent and interpret functions. ● Calculate volume of cylinders, cones, and spheres.	Unit 6 ● Use scatter plots to model data.	● Nonlinear functions ● Cross-sections and volumes in context

Defining Functions

A function is a rule that assigns exactly one output to each possible input.

Examples	Non-Examples
Input: Name **Output:** First letter of that name (e.g., Sneha → S)	**Input:** Letter **Output:** A name beginning with that letter (e.g., S → Sora)
Input: Any number **Output:** Three more than the input (e.g., $7 \rightarrow 10$)	**Input:** Digit **Output:** A number whose last digit is the input (e.g., $7 \rightarrow 207$)

Here are some more **examples** of functions:

$$y = 4 - 3x$$

Input	Output
-2	4π
-1	1π
0	0
1	1π
2	4π
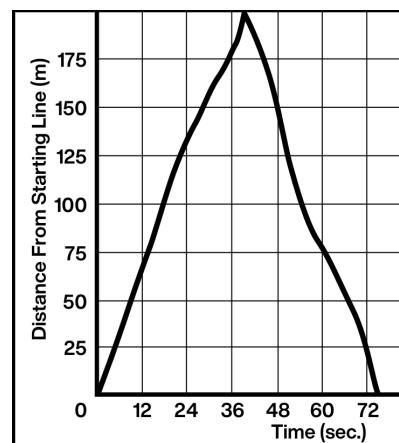
Representing and Interpreting Functions

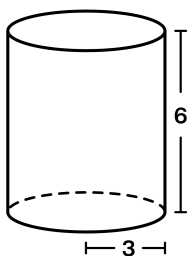A function can represent a story. Here is one example:
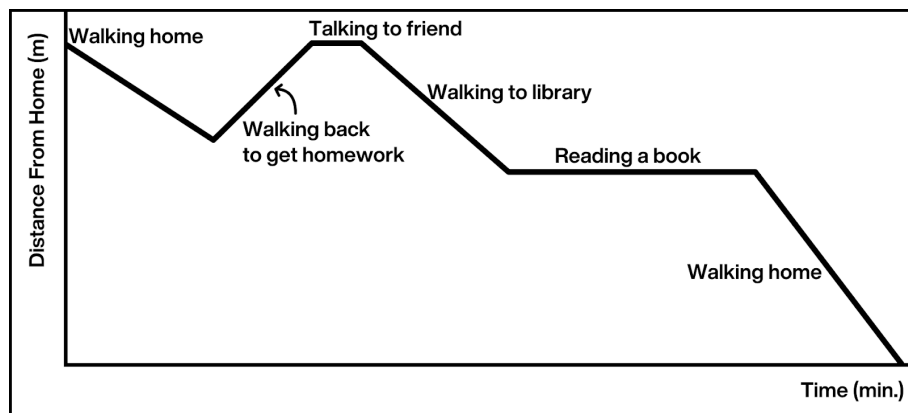
Independent Variable:
Time (min.)

Dependent Variable:
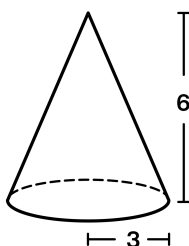Distance From Home (m)

Volume of Cylinders, Cones, and Spheres

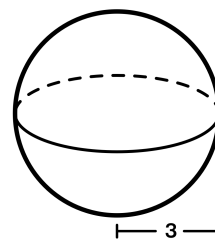Volume is the number of cubic units that fill a 3-D region without any gaps or overlaps.

Cylinder	**Cone**	**Sphere**

Cylinder

$$V = B \cdot h$$
$$= \pi r^2 \cdot h$$
$$= \pi \cdot (3)^2 \cdot (6)$$
$$= 9 \cdot 6 \cdot \pi$$
$$= 54\pi \text{ cubic units}$$

Cone

$$V = \frac{1}{3} B \cdot h$$
$$= \frac{1}{3} \pi r^2 \cdot h$$
$$= \frac{1}{3} \pi \cdot (3)^2 \cdot (6)$$
$$= \frac{1}{3} \cdot 9 \cdot 6 \cdot \pi$$
$$= 18\pi \text{ cubic units}$$

Sphere

$$V = \frac{4}{3} \pi r^2 \cdot r$$
$$= \frac{4}{3} \pi r^3$$
$$= \frac{4}{3} \pi \cdot (3)^3$$
$$= \frac{4}{3} \cdot 27 \cdot \pi$$
$$= 36\pi \text{ cubic units}$$

Try This at Home

Defining Functions

1.1   This table represents the total amount of data used compared to how many phone calls were made in a month.
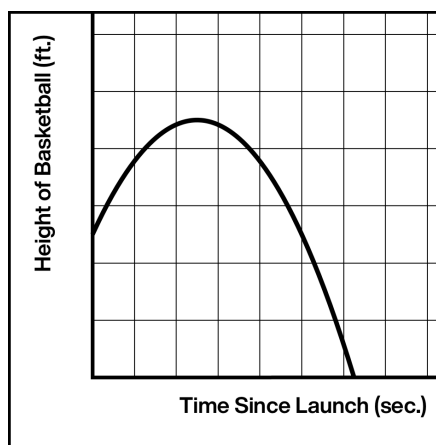
# of Phone Calls	Total Data Used (GB)
10	4.3
19	6.2
35	7.5
10	8.3

a.   Name the independent variable (input) and dependent variable (output).

b.   Decide whether the situation represents a function or not. Explain your thinking.

1.2   This graph represents the height of a basketball over time.

a.   Name the independent variable (input) and dependent variable (output).

b.   Decide whether the situation represents a function or not. Explain your thinking.

1.3   Brown rice costs 2 per pound. Beans cost 1.60 per pound. Jamar has 10 to spend to make a large meal of beans and rice for a potluck dinner. The amount of brown rice he can buy, r, is related to the amount of beans he can buy, b.

a.   Name the independent variable (input) and dependent variable (output).

b.   Decide whether the situation represents a function or not. Explain your thinking.
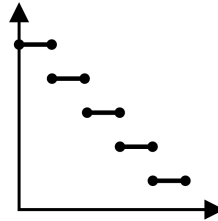
Representing and Interpreting Functions

Match each of the following situations with a graph (you can use a graph multiple times). Name the independent and dependent variables.
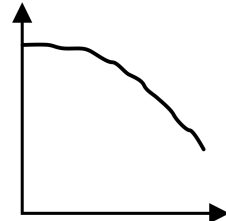
A.
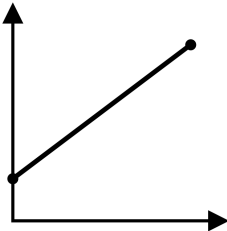
B.

C.

2.1 Daeja takes a handful of popcorn out of the bag every 5 minutes.

2.2 A plant grows the same amount every week.

2.3 The day started very warm, but then it slowly got colder.

2.4 A cylindrical glass sits on a counter.
The more water you pour in, the higher the water level is.

3. Write an equation in the form $y = mx + b$ that could represent the plant's growth. Explain what each number means in terms of the situation.

Volume of Cylinders, Cones, and Spheres

This cylinder has a height and radius of 5 cm.
Express your answers in terms of π .
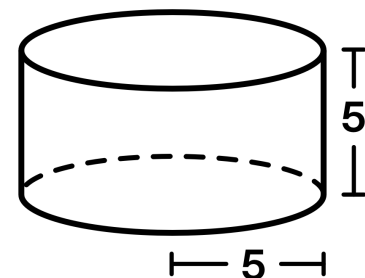
4.1 What is the diameter of the base?

4.2 What is the area of the base?

4.3 What is the volume of the cylinder?

4.4 What would the volume of a cone with the same height and radius be?

4.5 What would the height be if the volume of the cylinder remained the same, but the radius doubled?

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Solutions:

1.1a The *independent variable* represents the input of a function. The *dependent variable* represents the output of a function. Here, the independent variable is the number of phone calls; the dependent variable is the total data used.

1.1b This relationship is **not** a function because the number of calls does not uniquely determine the amount of data. For example, 10 phone calls results in both 4.3 GB and 8.3 GB of data.

1.2a By convention, the *independent variable* is represented on the horizontal axis and the *dependent variable* on the vertical axis. The independent variable in this situation is the time since launch. The dependent variable is the height of the basketball.

1.2b This relationship is a function because there is exactly one height for each time.

1.3a It is possible for either variable to be the independent variable. In this case, we are wondering about how much rice can be bought, so the independent variable is the amount of brown rice purchased. The dependent variable is the amount of beans purchased.

1.3b This relationship is a function because for every amount of beans, there is only one possible amount of rice Lin can buy if he wants to spend exactly $\$10$.

2.1 Graph B, Independent variable = Time (minutes), Dependent variable = Amount of popcorn left in bag

2.2 Graph A, Independent variable = Time (weeks), Dependent variable = Height of the plant

2.3 Graph C, Independent variable = Time (hours), Dependent variable = Temperature outside

2.4 Graph A, Independent variable = Volume of water poured in the glass, Dependent variable = Height of water in the glass

3. The equations vary. An example equation is $y = 2x + 5$, where 5 represents the height of the plant when you start measuring and 2 represents the number of inches the plant grows every week.

4.1 10 cm. The diameter is twice the length of the radius, and $2(5) = 10$.

4.2 25π cm². The area of a circle is π times the radius squared, or $(5)^2 \cdot \pi$.

4.3 125π cm³. The volume is the area of the base times the height. The area of the base here is 25π, so the volume is 125π cm³ since $25\pi \cdot 5 = 125\pi$.

4.4 $\dfrac{125\pi}{3}$ cm³. The volume of a cone is one-third the volume of the corresponding cylinder.

4.5 1.25 cm. If the radius doubled, then it would be 10 cm. There are various methods for finding the height. One method is to organize each quantity using a table. A sample table is shown below.

Radius (cm): 10

Base area (sq. cm): 100π

Height (cm): $\dfrac{125\pi}{100\pi} = 1.25$

Cylinder volume (cubic cm): 125π